



Analytical Solution of Magneto-Thermoelastic Diffusion Problem on a Hollow Cylinder

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In this article, an estimation is made to investigate the effect of magnetic field in generalized thermoelastic diffusion model in the context of Lord and Shulman theory in a perfectly conducting medium. The analytical solution in the Laplace transform domain is obtained by using the eigenvalue approach. The boundary conditions for the mechanical and Maxwell's stresses at the internal and outer surfaces are considered. An application of a hollow cylinder is investigated where the inner surface is traction free and subjected to thermal shock, while the outer surface is traction free and thermally isolated. The numerical results of displacement, temperature, concentration and stress as well as the chemical potential represented analytical and graphically.

Keywords: Eigenvalue Approach, Laplace Transform, Thermoelastic Diffusion, Magnetic Field.

1. INTRODUCTION

Biot¹ developed the coupled theory of thermoelastic to deal with a defect of the uncoupled theory that mechanical causes have no effect on the temperature. However, this theory shares a defect of the uncoupled theory in that it predicts infinite speeds of propagation for heat waves. During the second half of the twentieth century, non-isothermal problems of the theory of elasticity become increasingly important. This is due to their many applications in widely diverse fields. The first of such modeling is the extended thermoelasticity theory (LS) of Lord and Shulman,² who introduced the concept of thermal relaxation time into the classical Fourier law of heat conduction. The theory was extended for anisotropic body by Dhaliwal and Sherief.³

The theory of magneto-thermoelasticity is concerned with the influence of the magnetic field on the elastic and thermoelastic deformations of a solid body. This theory has aroused much interest in recent years, because of its application in various branches of science and technology. Electromagnetothermoelasticity investigates the interaction between temperature, strain, stress, and electromagnetic field in an elastic solid body. Recently, Refs. [4–33] and [34–51] have considered different problems by using the generalized thermoelasticity theories. Hetnarski and

Ignaczak⁵² presented a survey article of representative theories in the range of generalized thermoelasticity.

Diffusion can be defined as the random walk of an ensemble of particles, from regions of high concentration to regions of lower concentration. The studying of diffusion became increasingly important. This is due mainly to its many applications in geophysics and industrial applications. In integrated circuit fabrication, diffusion is used to introduce “dopants” in controlled amounts into the semiconductor substrate. In particular, diffusion is used to form the base and emitter in bipolar transistors, form integrated resistors, and form the source/drain regions in MOS transistors and dope poly-silicon gates in MOS transistors. The theory of thermoelastic diffusion is developed by Refs. [53–55]. In this theory, the coupled thermoelastic model is used. Reference [56] studied the Thermodiffusion in magnetic fluids by the modified separation theory. Reference [57] investigate the effect of thermal radiation on magnetohydrodynamics nanofluid flow and heat transfer by means of two phase model. The effects of heat transfer on peristaltic motion of Oldroyd fluid in the presence of inclined magnetic field is presented in Ref. [58]. Reference [59] developed the theory of generalized thermoelastic diffusion that predicts finite speeds of propagation for thermoelastic and diffusive waves. References [60–62] investigated the diffusion in a generalized thermoelastic solid in an infinite body with a cylindrical cavity while^{63,64}

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